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From spinors to forms: results on G-structures in supergravity and on topological field theories

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Abstract

This thesis is divided in two parts, that can be read separately even if both use the possibility of replacing spinors with differential forms in theories with supersymmetry.

The first part explores some recent results that have been obtained by applying the G-structure approach to type II supergravities. Using generalized complex geometry it is possible to reformulate the conditions for unbroken supersymmetry in type II supergravity in terms of differential forms. We use this result to find a classification for AdS_7 and AdS_6 solutions in type II supergravity.

Concerning AdS_7 solutions we find that in type IIB no solutions can be found, whereas in massive type IIA many new $\mathrm{AdS}_7 \times \mathcal{M}_3$ solutions are at disposal with the topology of the internal manifold $\mathcal{M}_3$ given by a three-sphere.

Concerning AdS_6 solutions, very few $\mathrm{AdS}_6 \times \mathcal{M}_4$ supersymmetric solutions are known in literature: one in massive IIA, and two IIB solutions dual to it. The IIA solution is known to be unique. We obtain a classification for IIB supergravity, by reducing the problem to two PDEs on a two-dimensional space Σ. $\mathcal{M}_4$ is given by a fibration of S^2 over Σ.

We also explore other two contexts in which the G-structure approach has revealed its usefulness: first of all we derive the conditions for unbroken supersymmetry for a Mink_2 (2,0) vacuum, arising from Type II supergravity on a compact eight-dimensional manifold $\mathcal{M}_8$. When $\mathcal{M}_8$ enjoys $\mathrm{SU}(4) \times \mathrm{SU}(4)$ structure the resulting system is elegantly rewritten in terms of generalized complex geometry. Finally we rewrite the equations for ten-dimensional supersymmetry in a way formally identical to an analogous system in N = 2 gauged supergravity; this provides a way to look for lifts of BPS solutions without having to reduce the ten-dimensional action.

The second part is devoted to study some aspects of two different Chern-Simons like theories: holomorphic Chern-Simons theory on a six-dimensional Calabi-Yau space and pure Chern-Simons theory on a Seifert three-dimensional manifold.

Concerning holomorphic Chern-Simons theory, we construct an action that couples the gauge field to off-shell gravitational backgrounds, comprising the complex structure and the (3,0)-form of the target space. Gauge invariance of this off-shell action is achieved by enlarging the field space to include an appropriate system of Lagrange multipliers, ghost and ghost-for-ghost fields. From this reformulation it is possible to uncover a twisted supersymmetric algebra for this model that strongly constrains the anti-holomorphic dependence of physical correlators.

Concerning pure Chern-Simons theory, we will show that the supersymmetric extension of this theory can be obtained without adding any additional spinorial fields by hand but simply by coupling the system to three-dimensional topological gravity. This

approach allows a more transparent classification of supersymmetric backgrounds, an easier way to understand the dependence of the theory from the geometrical backgrounds and finally gives an interpretation of such a dependence in terms of a topological anomaly.

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